PHY 102: DYNAMICS AND ELASTICITY

Chapter:1

Gravitation

Prof. Dr. N. S. Patil

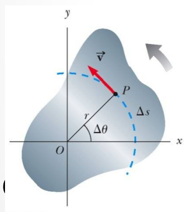
- Angular velocity, acceleration

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\theta_f - \theta_i}{t_f - t_i} \qquad \alpha = \frac{\omega_f - \omega_i}{t}$$

- Rotational/ Linear analogy

$$\Rightarrow \quad \Delta\theta \leftrightarrow \Delta x, \quad \omega_0 \leftrightarrow v_0, \quad \omega_f \leftrightarrow v_f, \quad \alpha \leftrightarrow a, \quad t \leftrightarrow t$$

- (angle in rac

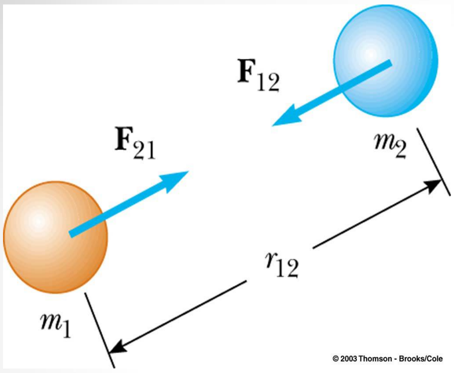


$$\Delta s = r\Delta\theta$$
$$v_t = r\omega$$
$$a_t = r\alpha$$

- (celeration. (to center)

$$a_{cent} = \omega^2 r = \frac{v^2}{r}$$

Prof. Dr. N. S. Patil



$$F = G\frac{m_1 m_2}{r^2}$$

$$G = 6.67 \times 10^{-11}\left(\frac{\text{m}^3}{kg \cdot s^2}\right)$$

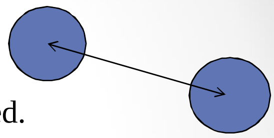
Newton's Law of Universal Gravitation

- Always attractive
- Proportional to both masses
- Inversely proportional to separation squared

Prof. Dr. N. S. Patil

Characteristics of the Gravitational Force

$$\vec{F} = G\frac{m_1 m_2}{r^2}$$



•The force is always attractive.

•There is a Newton's third law force pair involved.

•It acts along a line connecting the centers of the two objects (called a **Central Force**)

•It is inversely proportional to r^2 (called a "one over r squared" force)

•Experimental measurement show us that it is a conservative force (the gravitational force on earth is conservative-remember? This is a general expression of that same force)

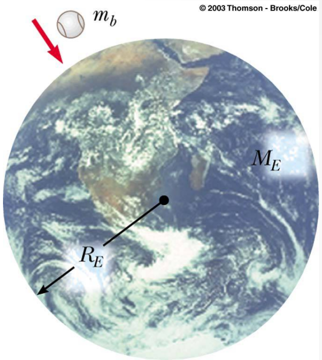
Prof. Dr. N. S. Patil

Weight

- Force of gravity on Earth

$$F_g = \frac{GM_E m}{R_E^2}$$

- But we know $F_g = mg$



$$\Rightarrow \quad g = \frac{GM_E}{R_E^2}$$

Prof. Dr. N. S. Patil

Defining the Potential Energy Associated with this Force

$$\Delta U = U_b - U_a = -W_{ab}$$

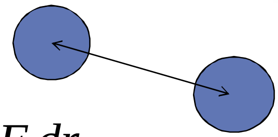
$$W_{ab} = \int_a^b \mathbf{F} \cdot d\mathbf{s} = \int_a^b F\,dr$$

$$\vec{F} = G\frac{m_1 m_2}{r^2}$$

Prof. Dr. N. S. Patil

POTENTIAL ENERGY

- Choose $U = 0$ at $r = \infty$



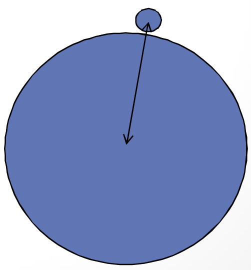
$$U = -\int_{\infty}^{r} F\, dr$$

$$U = -\frac{GMm}{r}$$

Prof. Dr. N. S. Patil

Gravitational Potential Energy Near Earth

$$U = -\frac{GMm}{R}$$



Prof. Dr. N. S. Patil

Example

Often people say astronauts feel weightless, because there is no gravity in space.

Prof. Dr. N. S. Patil

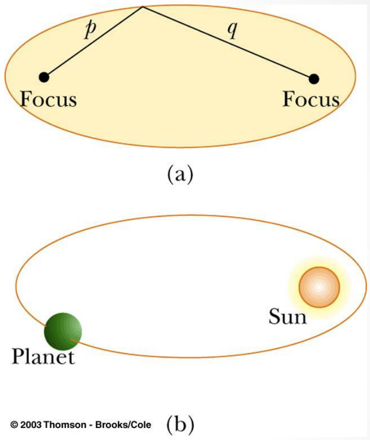
Kepler's Laws

1) Planets move in elliptical orbits with Sun at one of the focal points.
2) Line drawn from Sun to planet sweeps out equal areas in equal times.
3) The square of the orbital period of any planet is proportional to cube of the average distance from the Sun to the planet.

Prof. Dr. N. S. Patil

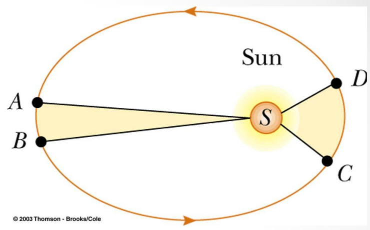
Kepler's First Law

- Planets move in elliptical orbits with the Sun at one focus.
- Any object bound to another by an inverse square law will move in an elliptical path
- Second focus is empty



Kepler's Second Law

- Line drawn from Sun to planet will sweep out equal areas in equal times
- Area from A to B equals Area from C to D.



True for any central force due to angular momentum conservation (next chapter)

Kepler's Third Law

- The square of the orbital period of any planet is proportional to cube of the average distance from the Sun to the planet.

$$\frac{R^3}{T^2} = \text{Constant}$$

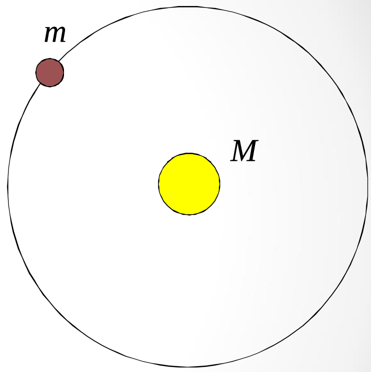
- The constant depends on Sun's mass, but is independent of the mass of the planet

Prof. Dr. N. S. Patil

Derivation of Kepler's Third Law

$$F_{\text{grav}} = \frac{GMm}{R^2} = ma_{\text{cent}} = m\omega^2 R$$

$$\omega = \frac{2\pi}{T}$$



$$\Rightarrow \frac{R^3}{T^2} = \frac{GM}{4\pi^2}$$

Prof. Dr. N. S. Patil

ENERGY IN CIRCULAR ORBITS

$$K = \tfrac{1}{2} m v^2 = \tfrac{1}{2} m \frac{GM}{r}$$

$$K = \frac{GMm}{2r}$$

$$U = -\frac{GMm}{r}$$

$$E = U + K = -\frac{GMm}{2r}$$

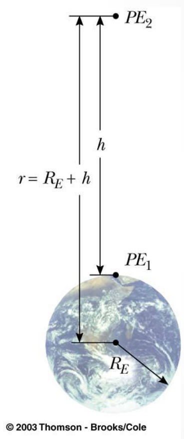
Prof. Dr. N. S. Patil

Gravitational Potential Energy

- PE = mgh valid only near Earth's surface
- For arbitrary altitude

$$PE = -G\frac{Mm}{r}$$

- Zero reference level is at r=∞



Torque

- Torque, τ, is tendency of a force to rotate object about some axis

$$\tau = Fd$$

- o F is the force
- o d is the *lever arm* (or moment arm)
- o Units are Newton-meters

Thank you